1 a $\dfrac{6(x+3)-6x}{x(x+3)} = \dfrac{18}{x(x+3)}$

b
$$\begin{aligned}\frac{18}{x(x+3)} &= 1\\ \frac{18-x(x+3)}{x(x+3)} &= 0\\ 18-x(x+3) &= 0\\ 18-x-3x &= 0\end{aligned}$$

Re-arrange and divide by -1:

$$\begin{aligned}x^2+3x-18 &= 0\\ (x-3)(x+6) &= 0\\ x = 3 &\text{ or } x=-6\end{aligned}$$

2
$$\begin{aligned}\frac{300}{x+5} &= \frac{300}{x}-2\\ 300x &= 300(x+5)-2x(x+5)\\ 300x &= 300x+1500-2x^2+10x\\ 2x^2-10x-1500 &= 0\\ x^2-5x-750 &= 0\\ (x+25)(x-30) &= 0\\ x = 25 &\text{ or } x=-30\end{aligned}$$

3 Let the numbers be n and $n+2$.

$$\begin{aligned}\frac{1}{n}+\frac{1}{n+2} &= \frac{36}{323}\\ \frac{1}{n}+\frac{1}{n+2}-\frac{36}{323} &= 0\\ \frac{323(n+2)+323n-36n(n+2)}{323n(n+2)} &= 0\\ 323n+646+323n-36n^2-72n &= 0\end{aligned}$$

Re-arrange and divide by -1:

$$\begin{aligned}36n^2-574n-646 &= 0\\ 18n^2-287-323 &= 0\\ (n-17)(18n+19) &= 0\\ n &= 17\end{aligned}$$

The numbers are 17 and 19.

4 a $\dfrac{40}{x}$

b $\dfrac{40}{x-2}$

c
$$\begin{aligned}\frac{40}{x-2}-\frac{40}{x} &= 1\\ 40x-40(x-2) &= x(x-2)\\ 80 &= x^2-2x\\ x^2-2x-80 &= 0\\ (x-10)(x+8) &= 0\\ \therefore x &= 10\end{aligned}$$

5 a $\text{Car} = \dfrac{600}{x}\text{ km/h; Plane} = \dfrac{600}{x}+220\text{ km/h}$

b Since the plane takes $x - 5.5$ hours to cover 600 km its average speed is also given by $\frac{600}{x-5.5}$. Hence:

$$\begin{aligned}\frac{600}{x} + 220 &= \frac{600}{x-5.5}\\ 600(x-5.5) + 220x(x-5.5) &= 600x\\ 600x - 3300 + 220x^2 - 1210x &= 600x\\ 220x^2 - 1210x - 3300 &= 0\\ 2x^2 - 11x - 30 &= 0\\ (2x-15)(x+2) &= 0\\ x &= 7.5\ (x > 0)\end{aligned}$$

$$\text{Average speed of car} = \frac{600}{7.5} = 80\text{ km/h}$$

$$\begin{aligned}\text{Average speed of plane} &= 80 + 220\\ &= 300\text{ km/h}\end{aligned}$$

6 $\text{Time taken by car} = \frac{200}{x}\text{ h}$

$$\text{Time taken by train} = \frac{200}{x+5}\text{ h} = \frac{200}{x} - 2\text{ h}$$

$$\begin{aligned}\frac{200}{x+5} &= \frac{200}{x} - 2\\ \frac{200}{x+5}\times x(x+5) &= \frac{200}{x}\times x(x+5) - 2\times x(x+5)\\ 200x &= 200(x+5) - 2x(x+5)\\ &= 200x + 1000 - 2x^2 - 10x\end{aligned}$$

$$\begin{aligned}2x^2 + 10x - 1000 &= 0\\ x^2 + 5x - 500 &= 0\\ (x-20)(x+25) &= 0\\ x &= 20 \text{ since } x > 0\end{aligned}$$

7 Let his average speed be x km/h.

His time for the journey is $\frac{108}{x}$ h.

$$\begin{aligned}\frac{108}{x} - 4\frac{1}{2} &= \frac{108}{x+2}\\ 108\times 2(x+2) - 4\frac{1}{2}\times 2x(x+2) &= 108\times 2x\\ 216x + 432 - 9x^2 - 18x &= 216x\\ -9x^2 - 18x + 432 &= 0\\ x^2 + 2x - 48 &= 0\\ (x-6)(x+8) &= 0\\ x &= 6\\ &\text{since } x > 0\end{aligned}$$

His average speed is 6 km/h.

8 a $\text{Usual time} = \frac{75}{x}\text{ h.}$

$$\begin{aligned}\frac{75}{x} - \frac{18}{60} &= \frac{75}{x+12.5}\\ \frac{75}{x} - \frac{3}{10} &= \frac{75}{x+12.5}\\ 75(x+12.5) - 0.3x(x+12.5) &= 75x\\ 75x + 937.5 - 0.3x^2 - 3.75x &= 75x\\ -0.3x^2 - 3.75x + 937.5 &= 0\end{aligned}$$

Divide by 0.15:

$$\begin{aligned}2x^2 + 25x - 6250 &= 0\\(x - 50)(2x + 125) &= 0\\x &= 50\end{aligned}$$

b $\text{Average speed} = x + 12.5 = 62.5$

$\text{Time} = \dfrac{75}{62.5} = 1.2\text{ h},$

or 1 hour 12 minutes, or 72 minutes.

9 Let the speed of the slow train be x km/h.

$$\begin{aligned}\text{The slow train takes } 3\frac{1}{2} - \frac{10}{60} &= \frac{7}{2} - \frac{1}{6}\\&= \frac{20}{6}\\&= \frac{10}{3}\text{ hours longer.}\end{aligned}$$

Compare the times:

$$\begin{aligned}\frac{250}{x+20} + \frac{10}{3} &= \frac{250}{x}\\750x + 10x(x + 20) &= 750(x + 20)\\750x + 10x^2 + 200x &= 750x + 15\,000\\10x^2 + 200x - 15\,000 &= 0\\x^2 + 20x - 1500 &= 0\\(x - 30)(x + 50) &= 0\\x &= 30\end{aligned}$$

Slow train: 30 km/h

Fast train: 50 km/h

10 Let the original speed of the car be x km/h. Compare the times:

$$\begin{aligned}\frac{105}{x+10} &= \frac{105}{x} - \frac{1}{4}\\420x &= 420(x + 10) - x(x + 10)\\420x &= 420x + 4200 - x - 10x\\x^2 + 10x - 4200 &= 0\\(x - 60)(x + 70) &= 0\\x &= 60\text{ km/h}\end{aligned}$$

11 Let x min be the time the larger pipe takes, and C the capacity of the tank. Form an equation using the rates:

$$\begin{aligned}\frac{C}{x} + \frac{C}{x+5} &= \frac{C}{11\frac{1}{9}}\\\frac{C}{x} + \frac{C}{x+5} &= \frac{9C}{100}\\\frac{1}{x} + \frac{1}{x+5} &= \frac{9}{100}\\100(x + 5) + 100x &= 9x(x + 5)\\100x + 500 + 100x &= 9x^2 + 45x\\200x + 500 &= 9x^2 + 45x\\9x^2 - 155x - 500 &= 0\\(x - 20)(9x + 25) &= 0\\x &= 20 \text{ since } x > 0\end{aligned}$$

The larger pipe takes 20 min and the smaller pipe takes 25 min.

12 Let x min be the original time the first pipe takes, and y min be the original time the second pipe takes.

Let C be the capacity of the tank.

The original rates are $\frac{C}{x}$ and $\frac{C}{y}$.

The combined rate is $\frac{C}{x}+\frac{C}{y}$.

$$\text{Total time taken} = \text{capacity} \div \text{rate}$$

$$\begin{aligned} C \div \left(\frac{C}{x}+\frac{C}{y}\right) &= C \div \frac{Cy+Cx}{xy} \\ &= C \times \frac{xy}{Cx+Cy} \\ &= \frac{xy}{x+y} = \frac{20}{3} \end{aligned}$$

New rates are $\frac{C}{x-1}$ and $\frac{C}{y+2}$.

The combined rate is $\frac{C}{x-1}+\frac{C}{y+2}$.

$$\begin{aligned} C \div \left(\frac{C}{x-1}+\frac{C}{y+2}\right) &= C \div \frac{C(y+2)+C(x-1)}{(x-1)(y+2)} \\ &= C \times \frac{(x-1)(y+2)}{Cx+Cy+C} \\ &= \frac{(x-1)(y+2)}{x+y+1} = 7 \end{aligned}$$

Solve the simultaneous equations:

$$\begin{aligned} \frac{xy}{x+y} &= \frac{20}{3} \\ \frac{(x-1)(y+2)}{x+y+1} &= 7 \end{aligned}$$

Multiply both sides of the first equation by $3(x+y)$:

$$\begin{aligned} 3xy &= 20x+20y \\ 3xy-20y &= 20x \\ y(3x-20) &= 20x \\ y &= \frac{20x}{3x-20} \end{aligned}$$

Substitute into the second equation, after multiplying both sides by $x+y+1$:

$$\begin{aligned} (x-1)(y+2) &= 7x+7y+7 \\ (x-1)\left(\frac{20x}{3x-20}+2\right) &= 7x+\frac{140x}{3x-20}+7 \\ (x-1)\frac{20x+2(3x-20)}{3x-20} &= 7x+\frac{140x}{3x-20}+7 \\ (x-1)\frac{26x-40}{3x-20} &= 7x+\frac{140x}{3x-20}+7 \\ (x-1)(26x-40) &= 7x(3x-20)+140x+7(3x-20) \\ 26x^2-66x+40 &= 21x^2-140x+140x+21x-140 \\ 5x^2-87x+180 &= 0 \\ (5x-12)(x-15) &= 0 \\ x &= 2.4 \text{ or } x = 15 \end{aligned}$$

$$y = \frac{20x}{3x-20} < 0 \text{ if } x = 2.4$$

$$\therefore \quad x = 15$$

$$y = \frac{20 \times 15}{3 \times 15 - 20} = 12$$

The first pipe now takes one minute less, i.e. $15 - 1 = 14$ minutes.

The second pipe now takes two minutes more, i.e. $12 + 2 = 14$ minutes.

13 Let the average speed for rail and sea be $x + 25$ km/h and x km/h respectively.

$$\text{Time for first route} = \frac{233}{x+25} + \frac{126}{x} \text{ hours.}$$

$$\text{Time for second route} = \frac{405}{x+25} + \frac{39}{x} \text{ hours.}$$

$$\frac{233}{x+25} + \frac{126}{x} = \frac{405}{x+25} + \frac{39}{x} + \frac{5}{6}$$

$$233 \times 6x + 126 \times 6(x+25) = 405 \times 6x + 39 \times 6(x+25) + 5x(x+25)$$

$$1398x + 756x + 18\,900 = 2430x + 234x + 5850 + 5x^2 + 125x$$

$$-5x^2 - 635x + 13\,050 = 0$$

$$x^2 + 127x - 2625 = 0$$

$$x = \frac{-127 + \sqrt{127^2 - 4 \times 1 \times 2625}}{2}$$

$$\approx 18.09$$

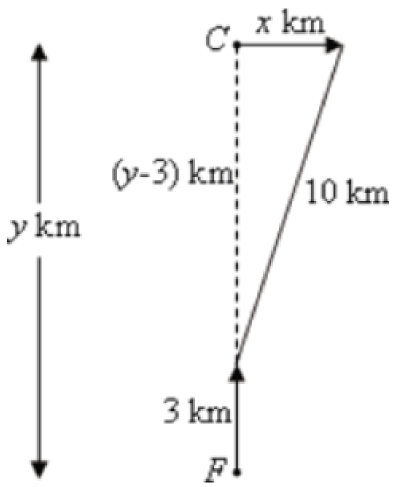
(Ignore negative square root as $x > 0$.)

Speed by rail is $18 + 25 = 43$ km/h and by sea is 18 km/h.

14 After 15 min, the freighter has travelled 3 km, bringing it to 12 km from where the cruiser was.

Let x km be the distance the cruiser has travelled in 15 minutes and y km the original distance apart of the ships.

The distance the cruiser has travelled can be calculated using Pythagoras' theorem.



$$x^2 + (y-3)^2 = 10^2 = 100$$

After a further 15 minutes, the distances will be $2x$ km and $(y - 6)$ km.

$$(2x)^2 + (y-6)^2 = 13^2$$

$$4x^2 + (y-6)^2 = 169$$

Multiply the first equation by 4 and subtract:

$$4(y-3)^2 - (y-6)^2 = 400 - 169$$

$$4y^2 - 24y + 36 - y^2 + 12y - 36 = 231$$

$$3y^2 - 12y - 231 = 0$$

$$y^2 - 4y - 77 = 0$$

$$(y-11)(y+7) = 0$$

$$y = 11$$

$$x^2 + 8^2 = 10^2$$

$$x = 6$$

The speed of the cruiser is $6 \div 0.25 = 24 \text{ km/h}$. The cruiser will be due east of the freighter when the freighter has travelled 11 km.

This will take $\frac{11}{12}$ hours. During that time the cruiser will have travelled $24 \times \frac{11}{12} = 22 \text{ km}$.

They will be 22 km apart.

15 Let x be the amount of wine first taken out of cask A.

After water is added, the concentration of wine in cask B is $\frac{x}{20}$.

If cask A is filled, it will receive x litres at concentration $\frac{x}{20}$.

The amount of wine in cask A will be $(20 - x) + x \times \frac{x}{20} = 20 - x + \frac{x^2}{20}$.

The concentration of wine in cask A will

be $\dfrac{20 - x + \frac{x^2}{20}}{20} = 1 - \frac{x}{20} + \frac{x^2}{400}$.

The amount of wine in cask B will be

$(20 - x) \times \frac{x}{20} = x - \frac{x^2}{20}$.

Mixture is transferred again.

The amount of wine transferred is

$\left(1 - \frac{x}{20} + \frac{x^2}{400}\right) \times \frac{20}{3} = \frac{20}{3} - \frac{x}{3} + \frac{x^2}{60}$.

Amount of wine in $A = \left(20 - x + \frac{x^2}{20}\right) - \left(\frac{20}{3} - \frac{x}{3} + \frac{x^2}{60}\right)$.

Amount of wine in $B = \left(x - \frac{x^2}{20}\right) + \left(\frac{20}{3} - \frac{x}{3} + \frac{x^2}{60}\right)$

$$\left(20 - x + \frac{x^2}{20}\right) - \left(\frac{20}{3} - \frac{x}{3} + \frac{x^2}{60}\right) = \left(x - \frac{x^2}{20}\right) + \left(\frac{20}{3} - \frac{x}{3} + \frac{x^2}{60}\right)$$

$$\begin{aligned} 20 - x + \frac{x^2}{20} - \frac{20}{3} + \frac{x}{3} - \frac{x^2}{60} &= x - \frac{x}{20} + \frac{20}{3} - \frac{x}{3} + \frac{x^2}{60} \\ -\frac{4x^2}{60} - \frac{4x}{3} + \frac{20}{3} &= 0 \\ \frac{x^2}{15} + \frac{4x}{3} - \frac{20}{3} &= 0 \\ x^2 + 20x - 100 &= 0 \\ (x - 10)^2 &= 0 \end{aligned}$$

10 litres was first taken out of cask A.